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X.—COMPARISON OF PRISMATIC AND DIFFRACTION SPECTRA.

By Professor E. C. Pickering.

Presented, June 9, 1875.

The object of the present communication is to afford a means of comparing the advantages of the two methods commonly employed for producing spectra, by diffraction gratings and by prisms. Two questions at once present themselves, the comparative length or dispersion, and the comparative brightness of the spectra. In adopting a standard of comparison, it is evidently necessary to select an absolute unit, which shall be wholly independent of the instrument employed, and defined entirely by the ordinary units of distance and direction. In comparing the two kinds of spectra, since an observing telescope and collimator are employed in both, it will be best first to compare the effect of the prisms and gratings alone, and then see how far both are affected by the telescopes. In the case of a diffraction grating, if i is the angle of incidence, r the angle of reflection, D the distance between the lines, λ the wave length, and n the order of the spectrum, these four quantities must be connected by the relation $n\lambda = D(\sin i + \sin r)$. The dispersion or angular deviation of two rays whose wave length differs by $d\lambda$ is found by differentiating r with regard to λ, recollecting that i being constant, its differential equals zero. We thus obtain $nd\lambda = D \cos r dr$, or $\frac{dr}{d\lambda} = \frac{n}{D \cos r}$. If now the grating is placed at right angles to the observing telescope, as in Meyerstein's spectrometer, the dispersion takes the very simple form $\frac{n}{D}$, or is independent of the angle of incidence and of the wave length, and hence is uniform throughout, and is simply proportional to the order of the spectrum, and inversely as the distance between the lines. This position has the further advantage that it gives a minimum of dispersion, and that consequently a slight error in setting is unimportant. If N is the number of lines per millimetre, or equals $\frac{1}{D}$, the dispersion assumes the still simpler form nN. This, then, forms the proper term of comparison for diffraction gratings, or the length of any minute portion of the spectrum will be proportional to its order, and to the number of lines per millimetre.

As an example, the grating that Angström employed in most of his measurements contained about 133 lines to the millimetre; and, as he commonly observed the fifth or sixth spectrum, his dispersion equalled 665 or 798. The admirable gratings of Mr. Rutherford contain 6480, 8640, 12,960, and 17,280 lines to an inch, or 255, 340, 510, and 680 lines to a millimetre. Accordingly the fifth spectrum of the 8640 grating would have a dispersion of 1700.

The case of refraction is a little more complex. As shown elsewhere (Proc. Am. Acad. vii. 478), when a beam of light, having an index of refraction n, passes through a prism having an angle a, we shall have the relation $\frac{dr_2}{dn} = \frac{\sin a}{\cos r_1 \cos r_2}$, in which r_1 and r_2 are the angles of refraction after passing the first and second surfaces. For the position of minimum of deviation, $r_1 = r_2 = \frac{1}{2}\,a$, and in this case $\frac{dr_2}{dn} = \frac{2 \sin \frac{1}{2} a}{\cos i} = \frac{2}{n} \text{ tang } i$. If, as is commonly the case, $a = 60°$, $\frac{dr_2}{dn} = \sec i$. But this gives $\frac{dr}{dn}$, while we want $\frac{di}{d\lambda}$, which may be obtained by multiplying by $\frac{dn}{d\lambda}$. The latter may be deduced from Cauchy's formula, $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$. Differentiating this equation, $\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3} - \frac{4C}{\lambda^5}$, and multiplying by $\frac{dr_2}{dn}$, gives $\frac{dn_2}{d\lambda} = \frac{-4}{n\lambda^3} \text{ tang } i \left(B + \frac{2C}{\lambda^2}\right)$, or for a 60° prism $-\frac{2}{\lambda^3}\left(B + \frac{2C}{\lambda^2}\right) \sec i$.

The substances most commonly used for spectroscope prisms are flint glass and bisulphide of carbon. The indices of refraction of the first of these varies very greatly with the composition, and that of the second with the temperature. The lines B, E, and G are selected as showing the effects of the ends and central portion of the spectrum. The indices for flint glass are those given by Fraunhofer for the specimen No. 23, and equal 1.62775, 1.64202, and 1.66028. For the bisulphide of carbon the temperature of 11°5 C. is employed, and the indices 1.6207, 1.6465, and 1.6886. These values give for the flint glass, $B = .00789$ and $C = .000307$. The corresponding values for the bisulphide are, $B = .00614$ and $C = .001972$, the wave lengths being expressed in thousandths of a millimetre. From these we may compute the three values of $\frac{dn}{d\lambda}$ for flint glass to be .0568, .1381, and .2804; and for bisulphide of carbon, .0818, .2293, and .6073. And finally multiplying these values by 1000 to change the unit from thousandths of a millimetre to millimetres, and by $\frac{dr}{dn}$, gives the following values for the dispersion of a 60° flint glass prism. For B, 98; for E, 242; and for G, 503. The corresponding values for a 60° prism filled with bisulphide of carbon are: for B, 140; for E, 404; and for G, 1133. Comparing these numbers with those given above for diffraction gratings, we see the superiority of the latter as regards dispersion, especially at the red end of the spectrum.

These advantages are, however, in a measure counterbalanced by the greater loss of light. It is shown elsewhere (Am. Jour. Sci. xlv.) that in a spectroscope containing ten 60° prisms the loss of light by reflection would equal 50.9 per cent; so that the transmitted ray would have an intensity of 49.1, the incident ray being taken as 100. This would be further reduced by the loss from absorption, but the amount would vary with the material, the wave length, and the length of path, or size of prisms. Estimating this loss as one half, still leaves the intensity of the whole of the spectrum as 25 per cent of the original beam passing through the slit. In a diffraction spectrum the light is much less; allowing one half for the light lost in the central white image, evidently if we have five spectra on each side, the average amount of light in each cannot exceed five per cent. And even this must be diminished by the loss due to reflection and absorption in the case of glass gratings, and to imperfect reflection in the case of speculum metal or silvered glass.

The discussion of the effect of the collimator and observing telescope on the dispersion involves another consideration; namely, the size of the image of the slit. To render this clearer, suppose we are observing the sodium spectrum, when a small amount of the metal is present.* We shall then obtain two sharply defined images of the slit separated by an interval dependent on the dispersion. These images may overlap, and will vary in width as the slit is open or shut, but their distances apart will not alter. Call w the true width of the slit, W that of its image as seen through the telescope, and referred to the distance of distinct vision, 250 mms., or ten inches. Also call c the focal length of the collimator, o that of the observing telescope, and e that of a lens equivalent to the eye-piece. Then $W = \frac{250\,wo}{ec}$. Again, the dispersion or interval between the two images will equal that of the prism or grating, multiplied by the magnifying power of the observing telescope, or $\frac{o}{e}$, and will be quite independent of the collimator.

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* If much sodium is present, the lines widen and become hazy, and with a large dispersion both appear again double, owing to the absorption of the outer layer of sodium vapor. This effect is readily obtained by putting a lump of borax in the flame, when at first it gives a bright blaze and shows the four lines; presently, however, the light becomes feeble, and the usual double line is alone seen. If now an image of the flame is projected on the slit, the spectrum of any part of it may be studied, and it will then be found that the central portion only gives the four lines, the edges giving the usual double line.

As in the microscope and telescope the highest powers are by no means those which give the best results, so in the spectroscope the best effects are not always obtained with the greatest dispersion. Increased angular dispersion is readily obtained by using a high power with the observing telescope, but the limit is soon reached, since the apparent width of the slit, and the various distortions, are increased in the same ratio. Moreover, with a very great dispersion the light is so far enfeebled that the spectrum becomes faint and the slit must be opened wider.

The most satisfactory test of the efficiency of a spectroscope, as of other optical instruments, is to examine some delicate object and compare the appearance with that obtained with other similar instruments. Formerly the D line was used for this purpose, which with any but the smallest instruments is seen to be double. In the solar spectrum it was soon found that there was a third line between these, and afterwards several other lines were noticed. The observations of Professor Cooke (Proc. Am. Acad. viii. 57), however, showed that most of these lines were due to the aqueous vapor in the earth's atmosphere, and that their visibility therefore depended very largely on the condition of the air. The E line is free from this objection; and, as it contains many more components, it furnishes a much more complete test. The following table gives the appearance of the E line as seen with various instruments. A dash denotes that the line opposite which it is drawn was visible. When a double line is seen as single, one of its components only is marked. The lines given in the map of Kirchhoff are shown in the column headed Kir. Those given by Angström are, in like manner, marked Ang. To obtain the lines seen with various instruments of the largest size, I asked several friends to draw all the lines they could see with their spectroscopes; and I take this occasion to express my thanks to them for the results. The column headed Sh. gives the results obtained by Mr. Sharples, with a large spectroscope belonging to Dr. Gibbs, having six 60° prisms, filled with chemically pure bisulphide of carbon.

The dispersion, therefore, as shown above, would be about 2400. Column Am. gives the lines seen by Dr. Amory, with a diffraction grating of Mr. Rutherford's, having 510 lines to a millimetre. As he employed the sixth spectrum, the dispersion was 3060. For the most recent and complete measurement I am indebted to Professor Young, who has measured the E line with a very perfect grating by Mr. Rutherford, having 340 lines to the millimetre ruled on silvered glass. As he used the eighth spectrum, the dispersion was 2720. These results have been taken as a basis, and the resultant wave lengths are given in the second column. My own observations are given in the column marked P., and were made in 1869 and 1870, with a spectroscope in which the light traversed each prism twice, giving a dispersion equivalent to 7, 9, or 11 flint glass prisms. This would correspond to dispersions of 1700, 2200, and 2640; but the best results were obtained with the two lower powers. All the observers used telescopes about a foot and a half in length.

We have thus four entirely independent maps, as neither observer had at the time a copy of the work of any of the others. The similarity of the results, with instruments differing so greatly in form and power, seems to show that we have nearly reached the limit beyond which an increase of dispersion is unadvisable; and as if with our largest instrument nearly all the lines really present in the spectrum were visible. It is much to be desired, however, that these lines may be compared with any other instruments of greater power, if such are ever constructed. It is only essential that the measurements should be made *before* comparison with the above results, since with the lines, as with faint stars, it is much easier to detect them when we know exactly where to look. Various tests may be selected from these lines for an instrument of any power. Thus to double the *E* lines, or to show 24 and 26 as two lines, is a good test for a one prism spectroscope of large size. The five pairs, (27, 28), (29, 30), (31, 32), (35, 36), and (37, 38), also form an excellent test for any but a very large instrument. The great number of double lines in this group, and more particularly in the *B* line, and in the electric spectrum of sodium, seems to prove most conclusively, as in the case of double stars, some real relation between the two components.

The last two columns give the relative intensity and width of the lines as estimated by Professor Young. Nos. 2, 16, and 27 are hazy, and No. 35 is a mere shade. The numbers under the observers' names give the approximate dispersion employed by each.

No.	Wave length.	Kir. 1400	Ang. 800	Sh. 2400	Am. 3060	P. 2000	Y. 2720	In-tensity.	Width.
1	5260.9	—	—	—	—	—	—	3	2
2	1.3	—		—	—	—	—	6	3
3	1.8			?	—	—	—	1	1
4	2.1						—	3	1.5
5	2.4	—	—	—	—	—	—	5	2.5
6	2.8			—	—	—	—	1	2.5
7	3.2						—	2.5	1.5
8	3.4	—	—	—	—	—	—	6	2
9	4.0			—	—		—	2	2
10	4.4			—	—	—	—	1.5	1
11	4.8			—		—	—	2	1.5
12	5.0	—	—	—	—	—	—	2	1.5
13	5.1			—		—	—	2	1.5
14	5.3			—	—	?			
15	5.6			—	—				
16	5.8	—	—	—	—	—	—	7	3.5
17	6.3						—	1	1
18	6.5			—	—	—	—	1	1
19	6.6						—	.5	1
20	7.0			—		—	—	1.5	1
21	7.5	—	—	—	—	—	—	2	1.5
22	7.7			—		—	—	1.5	1
23	8.3						—	.5	1
24	8.7	—	—	—	—	—	—	10	3.5
25	9.0			?		—	?	.5	2
26	9.5	—	—	—	—	—	—	8	2
27	69.9						?	.7	
28	70.3						—	.7	1
29	0.5			—	—	—	—	1.5	1
30	0.7			—		—	—	1	1
31	1.2			—	—	—	—	2	1.5
32	1.6			—		—	—	2	1.5
33	2.3			—		—	—	6	2
34	2.5	—	—	—	—	—	—	5	1.5
35	3.1						—	1	3
36	3.9			—	—	—	—	.7	1
37	4.2			—		—	—	1.5	1
38	4.4	—	—	—	—	—	—	2	1.5
39	5.0			—		—	—	2.5	1
40	5.3	—	—	—	—	—	—	4	2